Statistics Handbook

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All statistical tables were computed by the author.

Wilcoxon Rank-Sum Test

(Equivalent to the Mann-Whitney U test, except quicker, and the table has different values)

1. Rank all data, irrespective of group.
2. Calculate the sum of the ranks for the group with lower *n*. (If groups are of equal *n*, calculate the sum of the ranks for each group, and take the smaller).
3. The result is significant if your number is smaller or equal to the appropriate value in the tables below ($n_1 = n$ for smaller group, $n_2 = n$ for larger group).

Significance level = 0.05 (One-tailed)

n_1 \ n_2	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
6	28	30	32	34	35	37	39	41	43	44	46	48	50	52	53
7		39	41	43	46	48	50	52	54	57	59	61	63	66	68
8			52	54	57	60	62	65	67	70	73	75	78	81	83
9				66	69	72	75	78	81	84	87	90	94	97	100
10					83	86	90	93	96	100	103	107	110	114	117
11						101	105	109	112	116	120	124	128	132	136
12							121	125	130	134	138	142	147	151	155
13								143	148	152	157	162	166	171	176
14									167	172	177	182	187	192	197
15										192	198	203	209	215	220
16											220	226	232	238	244
17												249	256	262	269
18													281	287	294
19														314	321
20															349

Significance level = 0.025 (One-tailed)

n_1 \ n_2	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
6	26	28	29	31	32	34	36	37	39	40	42	44	45	47	48
7		37	39	40	42	44	46	48	50	52	54	56	58	60	62
8			49	51	53	56	58	60	63	65	67	70	72	75	77
9				63	65	68	71	74	76	79	82	85	87	90	93
10					79	82	85	88	91	94	97	100	104	107	110
11						96	100	103	107	110	114	117	121	124	128
12							116	119	123	127	131	135	139	143	147
13								137	141	145	150	154	159	163	167
14									160	165	169	174	179	184	188
15										185	190	195	200	205	211
16											211	217	223	228	234
17												240	246	252	258
18													271	277	283
19														303	310
20															337

Wilcoxon Matched-Pairs Test

1. Calculate the difference between each pair.
2. Remove pairs whose difference is zero, and reduce *n* accordingly.
3. Rank the differences of remaining pairs, ignoring their sign.
4. Calculate the sum of the ranks of the positive differences (T^+).
5. Calculate the sum of the ranks of the negative differences (T^-).
6. Let T be the smaller of T^+ and T^-.
7. The result is significant if T is smaller or equal to the appropriate value in the table below.

	Significance (1-tailed)	
N	*0.05*	*0.025*
6	2	1
7	4	2
8	6	4
9	8	5
10	11	8
11	14	10
12	18	14
13	21	17
14	26	21
15	31	25
16	36	30
17	41	35
18	47	40
19	54	46
20	60	52

Normal distribution

Z-test
To find the probability with which a score X comes from a normal distribution with a mean of μ and a standard deviation of σ.

1. Calculate:

$$z = \frac{X - \mu}{\sigma}$$

2. Ignore the sign of z. The table below gives the one-tailed probability.

Z-table

z	p	z	p	z	p	z	p
0.00	0.5000	1.52	0.0643	2.16	0.0154	2.76	0.0029
0.05	0.4801	1.54	0.0618	2.18	0.0146	2.78	0.0027
0.10	0.4602	1.56	0.0594	2.20	0.0139	2.80	0.0026
0.15	0.4404	1.58	0.0571	2.22	0.0132	2.82	0.0024
0.20	0.4207	1.60	0.0548	2.24	0.0125	2.84	0.0023
0.25	0.4013	1.62	0.0526	2.26	0.0119	2.86	0.0021
0.30	0.3821	1.64	0.0505	2.28	0.0113	2.88	0.0020
0.35	0.3632	1.66	0.0485	2.30	0.0107	2.90	0.0019
0.40	0.3446	1.68	0.0465	2.32	0.0102	2.92	0.0018
0.45	0.3264	1.70	0.0446	2.34	0.0096	2.94	0.0016
0.50	0.3085	1.72	0.0427	2.36	0.0091	2.96	0.0015
0.55	0.2912	1.74	0.0409	2.38	0.0087	2.98	0.0014
0.60	0.2743	1.76	0.0392	2.40	0.0082	3.00	0.0013
0.65	0.2578	1.78	0.0375	2.42	0.0078	3.02	0.0013
0.70	0.2420	1.80	0.0359	2.44	0.0073	3.04	0.0012
0.75	0.2266	1.82	0.0344	2.46	0.0069	3.06	0.0011
0.80	0.2119	1.84	0.0329	2.48	0.0066	3.08	0.0010
0.85	0.1977	1.86	0.0314	2.50	0.0062	3.10	0.0010
0.90	0.1841	1.90	0.0287	2.52	0.0059	3.12	0.0009
1.00	0.1587	1.92	0.0274	2.54	0.0055	3.14	0.0008
1.05	0.1469	1.94	0.0262	2.56	0.0052	3.16	0.0008
1.10	0.1357	1.96	0.0250	2.58	0.0049	3.18	0.0007
1.15	0.1251	2.00	0.0228	2.60	0.0047	3.20	0.0007
1.20	0.1151	2.02	0.0217	2.62	0.0044	3.22	0.0006
1.25	0.1056	2.04	0.0207	2.64	0.0041	3.24	0.0006
1.30	0.0968	2.06	0.0197	2.66	0.0039	3.26	0.0006
1.35	0.0885	2.08	0.0188	2.68	0.0037	3.30	0.0005
1.40	0.0808	2.10	0.0179	2.70	0.0035	3.50	0.0002
1.45	0.0735	2.12	0.0170	2.72	0.0033	3.75	0.0001
1.50	0.0668	2.14	0.0162	2.74	0.0031	4.00	0.0000

Related samples t-test

1. Calculate the difference (D) between each pair.
2. Calculate the mean of the differences, $\overline{D}$.
3. Calculate the standard deviation of the differences

$$s_D = \sqrt{\frac{\sum\left(D - \overline{D}\right)^2}{N - 1}}$$

4. Calculate the standard error of the differences:

$$s_{\overline{D}} = \frac{s_D}{\sqrt{N}}$$

5. Calculate the *t* statistic:

$$t = \frac{\overline{D}}{s_{\overline{D}}}$$

6. Ignore the sign of *t*. The result is significant if *t* is greater than the appropriate value in the *t*-table.

- For a related samples *t*-test with *N* pairs, *df = N-1*.

t-table

	Significance Level (2-tailed)			
df	*0.1*	*0.05*	*0.025*	*0.01*
1	6.314	12.706	25.452	63.656
2	2.920	4.303	6.205	9.925
3	2.353	3.182	4.177	5.841
4	2.132	2.776	3.495	4.604
5	2.015	2.571	3.163	4.032
6	1.943	2.447	2.969	3.707
7	1.895	2.365	2.841	3.499
8	1.860	2.306	2.752	3.355
9	1.833	2.262	2.685	3.250
10	1.812	2.228	2.634	3.169
11	1.796	2.201	2.593	3.106
12	1.782	2.179	2.560	3.055
13	1.771	2.160	2.533	3.012
14	1.761	2.145	2.510	2.977
15	1.753	2.131	2.490	2.947
16	1.746	2.120	2.473	2.921
17	1.740	2.110	2.458	2.898
18	1.734	2.101	2.445	2.878
19	1.729	2.093	2.433	2.861
20	1.725	2.086	2.423	2.845

Unrelated samples t-test

Equal N

1. Let N be the sample size of each group.
2. Calculate the mean for each group, $\overline{X}_1$ and $\overline{X}_2$
3. Calculate the standard deviation for each group, s_1 and s_2.

$$s = \sqrt{\frac{\sum\left(X - \overline{X}\right)^2}{N - 1}}$$

4. Calculate the t statistic

$$t = \frac{\left(\overline{X}_1 - \overline{X}_2\right)}{\sqrt{\frac{s_1^2 + s_2^2}{N}}}$$

5. Ignore the sign of t. The result is significant if t is greater than the appropriate value in the t-table.

- $df = 2N\text{-}2$, assuming equal variances. Where variances are different, df are no smaller than $N\text{-}1$.

Unequal N

1. Let N_1 and N_2 be the sample sizes of the two groups.
2. Calculate the mean for each group, $\overline{X}_1$ and $\overline{X}_2$
3. Calculate the standard deviation for each group, s_1 and s_2.

$$s = \sqrt{\frac{\sum\left(X - \overline{X}\right)^2}{N - 1}}$$

4. Calculate the pooled variance estimate:

$$s_p^2 = \frac{\left(N_1 - 1\right)s_1^2 + \left(N_2 - 1\right)s_2^2}{N_1 + N_2 - 2}$$

5. Calculate the t-statistic:

$$t = \frac{\overline{X}_1 - \overline{X}_2}{\sqrt{s_p^2\left(\frac{1}{N_1} + \frac{1}{N_2}\right)}}$$

6. Ignore the sign of t. The result is significant if t is greater than the appropriate value in the t-table.

- $df = N_1 + N_2 \text{-}2$, assuming equal variances. Where variances are different, df are no lower than the smaller of $(N_1 - 1)$ and $(N_2 - 1)$.

Variance test

1. Calculate the variance for each of the two groups:

$$s^2 = \frac{\sum\left(X - \overline{X}\right)^2}{N-1}$$

2. Let S_L^2 be the smaller of the two variances, and S_H^2 be the larger.
3. Calculate the F-ratio:

$$F = \frac{S_H^2}{S_L^2}$$

4. The variances are significantly different if *F* is greater than the appropriate value in the *F* table.

- The degrees of freedom for the numerator are $(N_H - 1)$, where N_H is the sample size for the group with higher variance. *df* for the denominator are $(N_L - 1)$.
- This is a two-tailed test. F tables give one-tailed significance levels.

F table

Degrees of Freedom for Denominator	**Significance Level = 0.025** Degrees of Freedom for Numerator											
	1	2	3	4	5	6	7	8	9	10	15	20
1	647.8	799.5	864.2	899.6	921.8	937.1	948.2	956.6	963.3	968.6	984.9	993.1
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.43	39.45
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.25	14.17
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.66	8.56
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.43	6.33
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.27	5.17
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.57	4.47
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.10	4.00
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.77	3.67
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.52	3.42
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.33	3.23
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.18	3.07
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.05	2.95
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	2.95	2.84
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.86	2.76
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.79	2.68
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.72	2.62
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.67	2.56
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.62	2.51

20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.57	2.46

Binomial Distribution

Let the outcomes be *P* and *Q*, and the number of trials be *N*. The probability of the *P* outcome occurring *exactly* *X* times is given by:

$$prob(X) = \frac{N!}{X!(N-X)!} p^X q^{(N-X)}$$

where

p = The probability of the *P* outcome
q = The probability of the *Q* outcome
$p+q = 1$

N! is **N factorial** e.g. 4! = 4 x 3 x 2 x 1 = 24.

Sign test

A simple test for the difference between two related groups is to calculate the difference for each pair of scores and then count the number of positive differences. If there is no significant difference between the groups, then the number of positive differences is described by a binomial distribution where $p = q = 0.5$.

Normal approximation to the Binomial distribution

Where *Np* and *Nq* are both greater than 5, the binomial distribution is approximately normal, with a mean of *Np* and a standard deviation of $\sqrt{Npq}$. A *Z*-test may be used in such situations.

Chi-square

- The formula for chi-square is:

$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

where O = observed frequency and E = expected frequency. Expected values depend what you're trying to do. Some examples:

<u>N-by-1 table</u>
There is one variable, with N levels. There are N observed frequencies, and the null hypothesis is that they do not differ. Expected frequency is the mean of the observed frequencies. Degrees of freedom = $N - 1$.

Example:

	Heads	**Tails**
Observed	83	17
Expected	50	50

<u>Contingency table</u>
There are two variables. One variable has M levels. The other variable has N levels. There are $N \times M$ observed frequencies. The null hypothesis is that the two variables are independent.

Expected frequency = Row Total x Column Total / Grand Total

Degrees of freedom = $(N-1)(M-1)$

Example (expected frequencies given in brackets)

	Non-smoker	Smoker	**TOTAL**	
Male	36 (26)	42 (52)	78	
Female	23 (33)	76 (66)	99	
TOTAL	59	118	177	⇐*Grand total*

<u>Model fit</u>
The null hypothesis is that expected frequencies do not differ from a particular prediction. Expected frequencies are known or can be calculated from available information.

Example:

	1st	**2:1**	**2:2**	**3rd**

Frequencies of degree class in final exam	18	67	5	0
Predictions of examiners prior to marking	9	45	30	6

There is no universal formula for degrees of freedom in a model-fit chi-square. It will depend on the way the expected frequencies have been derived. However, *df* are smaller than or equal to the number of predictions.

Test of significance

- The result is significant if the calculated value of chi-square exceeds the appropriate value on the chi-square table.
- Chi-square tests are generally performed as *multi-tailed* tests. The significance levels on the table given are appropriate for most common applications of chi-square.

Chi-square table

	Significance Level			
df	*0.1*	*0.05*	*0.025*	*0.01*
1	2.706	3.841	5.024	6.635
2	4.605	5.991	7.378	9.210
3	6.251	7.815	9.348	11.345
4	7.779	9.488	11.143	13.277
5	9.236	11.070	12.832	15.086
6	10.645	12.592	14.449	16.812
7	12.017	14.067	16.013	18.475
8	13.362	15.507	17.535	20.090
9	14.684	16.919	19.023	21.666
10	15.987	18.307	20.483	23.209
11	17.275	19.675	21.920	24.725
12	18.549	21.026	23.337	26.217
13	19.812	22.362	24.736	27.688
14	21.064	23.685	26.119	29.141
15	22.307	24.996	27.488	30.578
16	23.542	26.296	28.845	32.000
17	24.769	27.587	30.191	33.409
18	25.989	28.869	31.526	34.805
19	27.204	30.144	32.852	36.191
20	28.412	31.410	34.170	37.566

Correlation

Calculation of the correlation co-efficient r between two variables x and y (*Pearson product-moment correlation*)

1. Calculate the standard deviation of x:

$$s = \sqrt{\frac{\sum\left(X - \overline{X}\right)^2}{N-1}}$$

2. Calculate the standard deviation of y

3. Calculate the *co-variance* of x and y:

$$\text{cov}_{XY} = \frac{\sum\left(X - \overline{X}\right)\left(Y - \overline{Y}\right)}{N-1}$$

4. Calculate r

$$r = \frac{\text{cov}_{XY}}{s_X s_Y}$$

5. The correlation is significant if r exceeds the appropriate value on the table below.

- Spearman's rank-order correlation co-efficient r_S can be calculated by applying the above procedure to the ranks of x and y, instead of the raw scores. **Rank *x* and *y* separately.**
- In the context of correlation, "tails" refers to the sign of r. If the test is for r simply being different from zero, then the test is two-tailed.

Linear Regression

Procedure for finding the best-fitting line of the form $y = \mathbf{b}x + \mathbf{a}$

1. Calculate the gradient of the best-fitting straight line:

$$b = \frac{\text{cov}_{XY}}{s_X^2}$$

2. Calculate the intercept of that line:

$$a = \overline{Y} - b\overline{X}$$

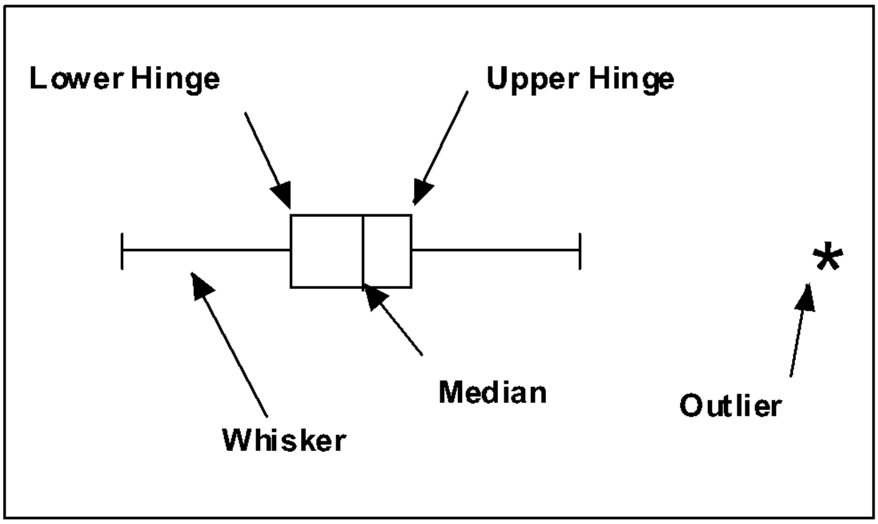
3. If the correlation co-efficient r is significantly different from zero, then b is significantly different from zero.

Critical Values of r (and r_S)

	Two-tailed significance Level		
N	*0.1*	*0.05*	*0.01*
9	0.575	0.657	0.785
10	0.544	0.625	0.754
11	0.517	0.596	0.726
12	0.494	0.571	0.701
13	0.473	0.549	0.677
14	0.455	0.529	0.656
15	0.439	0.511	0.637
16	0.424	0.495	0.619
17	0.411	0.480	0.602
18	0.399	0.467	0.587
19	0.388	0.454	0.572
20	0.377	0.442	0.559

The values given in the table are based on an approximation that is accurate to within 0.02 over the range covered.

Box Plot



Median position = (N+1) / 2
Lower hinge position* = (median position + 1) / 2

Upper hinge position = N + 1 - lower hinge position

Inter-quartile range (IQR) : Difference between data values at upper and lower hinge positions
Whisker = 1.5 x IQR
Outliers: Typically, points more than two whiskers from the nearest hinge.

** Ignore fractional component of median position in calculation of lower hinge position.*