Statistics by Hand

An Introductory Course for Psychologists

Return of the t-test

Version 3.0

Reminder: Related samples *t*-test

- 1. Calculate the difference between each pair.

Auditory RT	230	300	250	240	330	260	280	250	300	270
Visual RT	260	230	220	250	220	260	210	310	270	240
	-30	70	30	-10	110	0	70	-60	30	30
Auditory RT	320	240	320	330	280	290	280	310	330	320
Visual RT	270	260	210	390	200	290	310	280	170	260
	50	-20	110	-60	80	0	-30	30	160	60

- 2. Calculate the mean difference

Reminder: Related samples *t*-test

Difference	X - mean	(X - mean)2
-30	-61	3721
70	39	1521
30	-1	1
-10	-41	1681
110	79	6241
0	-31	961
70	39	1521
-60	-91	8281
30	-1	1
30	-1	1
50	19	361
-20	-51	2601
110	79	6241
-60	-91	8281
80	49	2401
0	-31	961
-30	-61	3721
30	-1	1
160	129	16641

Mean	31
Sum (X - mean)2	65980
N-1	19
Variance	3472.6316
Std. Dev.	58.929039

3. Calculate the standard deviation of the differences.
4. Calculate the standard error:

standard error = s / √N

= 58.93 / √20 = 13.2

Reminder: Related samples *t*-test

- 5. Divide the mean difference by the standard error.

  t = 31 / 13.2 = 2.35

- 6. Calculate d.f. (= N-1)
- 7. If *t exceeds* the appropriate value in the table then the result is significant.

Unrelated samples t-test

- Just like Wilcoxon, there are between-subjects and within-subjects versions of the t-test

- Similar procedure to related t-test
  - except there are now 2 means and 2 s.d.
  - Still based on CLT, but more complicated to demonstrate … so I won't.

Unrelated samples t-test

	Helpful	X - mean	(X - mean)2		Misleading	X - mean	(X - mean)2
	57.42	4.57	20.89		67.10	14.25	203.01
	70.61	17.76	315.43		34.06	- 18.79	353.05
	36.59	- 16.26	264.51		42.25	- 10.60	112.42
	53.86	1.01	1.02		33.87	- 18.98	360.38
	47.46	- 5.39	29.01		52.11	- 0.74	0.55
	46.67	- 6.18	38.19		70.32	17.47	305.14
	46.84	- 6.01	36.12		84.43	31.58	997.12
	63.35	10.50	110.17		83.45	30.60	936.50
Mean	52.85				58.45		
Sum (X - mean)2			815.33				3,268.18
N-1			7.00				7.00
Variance			116.48				466.88

1. Calculate mean & variance for each group

Unrelated samples t-test

2. Calculate the difference between the means:

$$\overline{X}_1 - \overline{X}_2 = 58.45 - 52.85 = 5.6$$

3. Calculate the standard error (defined differently for unrelated t-test):

$$\sqrt{\frac{s_1^2 + s_2^2}{N}} = \sqrt{\frac{116.48 + 466.88}{8}} = 8.54$$

N = Sample size of each group

Unrelated samples t-test

$$t = \frac{\overline{X}_1 - \overline{X}_2}{s.e.} = \frac{5.6}{8.52} = 0.66$$

4. Calculate *t*
5. Calculate d.f.
   - *d.f. = 2N - 2 = 14*

- Critical value is 2.131
- No significant difference

Unrelated samples t-test

2. Calculate the difference between the means:

What if N for the two groups is different?

$$\bar{X}_1 - \bar{X}_2$$

3. Calculate the standard error (defined differently for unrelated t-test):

$$\sqrt{\frac{s_1^2 + s_2^2}{N}} = \sqrt{\frac{116.48 + 466.88}{8}} = 8.54$$

N = Sample size of each group

Pooled variance estimate

- Where sample size unequal, the larger sample should contribute proportionately more to the calculation of standard error.
- This is done via the pooled variance estimate. The appropriate equation is:

$$s_p^2 = \frac{(N_1 - 1)s_1^2 + (N_2 - 1)s_2^2}{N_1 + N_2 - 2}$$

Unequal N procedure

1. Calculate mean and variance for each group.
2. Calculate difference between means
3. Calculate the pooled variance estimate:

$$s_p^2 = \frac{(N_1 - 1)s_1^2 + (N_2 - 1)s_2^2}{N_1 + N_2 - 2}$$

Unequal N procedure

4. Use the pooled variance estimate to calculate the standard error, taking into account the differing sample sizes:

$$\sqrt{s_p^2\left(\frac{1}{N_1}+\frac{1}{N_2}\right)}$$

Unequal N procedure

$$t = \frac{\overline{X}_1 - \overline{X}_2}{s.e.}$$

5. Calculate *t*
6. Calculate d.f.
   - *d.f.* = $N_1 + N_2 - 2$

Reminder: t-test or Wilcoxon?

- Use a t-test if...
  - N is large (more than 30)
    - (because then CLT part 3 will be correct)
  - or, you know the population is roughly normal (symmetrical with only one peak)
    - Use histogram to assess for each group
    - N.B.: histogram can't be done for N<10
  - Otherwise..
    - Use a Wilcoxon

t-test or Wilcoxon: Additional

- In a unrelated-samples t-test, use of t-tables only valid if the two populations are of equal variance.
- Variance test required to see whether this assumption is violated.
- If significant, assumption violated
  - Use Wilcoxon rank-sum
- If n.s., assumption might be OK
  - Use unrelated t-test if other conditions met

Reminder: Variance test

	Helpful	X - mean	(X - mean)2		Misleading	X - mean	(X - mean)2
	57.42	4.57	20.89		67.10	14.25	203.01
	70.61	17.76	315.43		34.06	- 18.79	353.05
	36.59	- 16.26	264.51		42.25	- 10.60	112.42
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Mean	52.85				58.45		
Sum (X - mean)2			815.33				3,268.18
N-1			7.00				7.00
Variance			116.48				466.88

Variance test:

Divide larger variance by smaller

F = 466.88 / 116.48 = 4.01

If F exceeds appropriate value in F-table, difference is significant.

Problems

- *T-3*

- *T-4*